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Allocation of Borders of Objects and Definition of the Sizes Imperfections in Compton Tomographies

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Abstract

At the control over the help of back - absent-minded radiation the information on the linear sizes heterogeneity contains in functional dependence of change of number of quantums in detector N_s at change of position of the disseminating volume V concerning heterogeneity on a direction of scanning.
In article the behavior of scanning function $N_S(z)$ on one direction z is investigated provided that the quantity of quantums in the detector is unequivocally defined{determined} by the size of the disseminating volume (DV) last in heterogeneity. The function changing only at transition through border of heterogeneity is formulated, influence of easing of primary and absent-minded beams by the previous layers, and also deformation of aperture function of the disseminating volume is investigated due to easing in itself DV.

Mutual moving DV and heterogeneity is a basis of reception of the information on internal structure of a controllable product. Schematically it can be submitted Fig. 1. The disseminating volume is the geometrical figure formed by crossing of a primary beam of radiation n0 which is formed by collimator a source and a beam of absent-minded radiation ns which shape is defined by collimator's geometry of receiver (figure DEFG). The quantity of unitary absent-minded quantums in the detector in position DV corresponding to coordinate z, is defined by size of element ABF which is taking place in heterogeneity V(z).

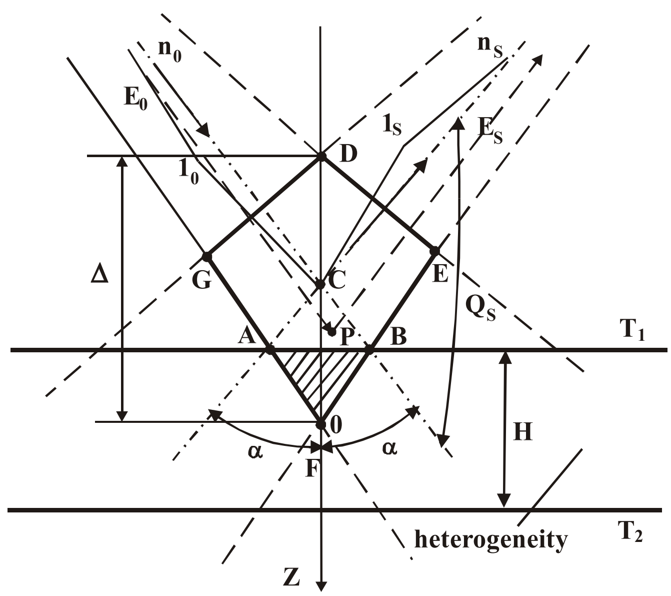

Fig. 1. Geometry of scanning

The size of the disseminating volume *V(Z)* is constant at its movement inside object, but its full reflective ability changes at transition through borders of heterogeneity T1 and T2 (Z changes from $-\Delta$ up to 0 for forward border of heterogeneity T1 and from up to $H-\Delta$ for *H* back border T2). At movement of volume V inside heterogeneity (i.e. at $0<Z<H-\Delta$ number of quantums in the detector decreases exponentially and does not bear the information on position of borders T1 and T2 Fig. 1. Hence, for definition H it is necessary to formulate the algorithm allocating sites of transition of borders T1 and T2, i.e. to formulate scanning function *B(z)* which would be equal to zero at movement inside heterogeneity and allocated only its borders. We shall count geometry symmetric concerning a direction of scanning Z. Then it is possible to write down $l_0 = l_S = \frac{z}{\cos\alpha}$ and the derivative $\frac{\partial N_S(Z)}{\partial Z}$ on a site $0<Z<H-\Delta$ will be equal

$$\frac{\partial N_S(Z)}{\partial Z} = -A\cdot V\frac{\mu_0+\mu_S}{\cos\alpha}\exp\left[-\left(\mu_0+\mu_S\right)\frac{Z}{\cos\alpha}\right] \tag{1}$$

It too an exhibitor, only having negative values and distinguished from $N_s(z)$ on a multiplier $\frac{\mu_0+\mu_S}{\cos\alpha}$. Hence, if a derivative $\frac{\partial N_S(Z)}{\partial Z}$ to increase on factor $K = \frac{\cos\alpha}{\mu_0+\mu_S}$ and to combine with $N_S(Z)$, on a site $0<Z<H-\Delta$ such sum will give 0.

$$B(Z) = N_S(Z) + K\frac{\partial N_S(Z)}{\partial Z} = 0 \tag{2}$$

On sites of transition through forward and back borders

On borders T_1 and T_2 $B(Z)=\left[V(Z)+\frac{\cos\alpha}{\mu_0+\mu_S}\frac{\partial V(Z)}{\partial Z}-V(Z)\right]\cdot\exp^{(-\mu_0+\mu_S)\cdot\frac{z}{\cos\alpha}}$, or

$$B(Z) = A\frac{\cos\alpha}{\mu_0+\mu_S}\frac{\partial V(Z)}{\partial Z}e^{-\left(\mu_0+\mu_S\right)\frac{Z}{\cos\alpha}} \tag{3}$$

For the set geometry and energy of quantums *Eo* size $A\frac{\cos\alpha}{\mu_0+\mu_S} = const = C$.

Then scanning function *B(z)* can be written down as

$$B(Z) = C\frac{\partial V(Z)}{\partial Z}e^{-\frac{\mu_0+\mu_S}{\cos\alpha}\cdot Z} \tag{4}$$

For studying character of function *B(z)* on sites of an entrance and an exit of the disseminating volume it is necessary to take into account, that for reception of the information on back border of heterogeneity T2, absent-minded quantums from the bottom layers of heterogeneity should get in the detector. Hence, condition $H<\lambda_S$ (where λ_S - length of free run of absent-minded quantums) should satisfy. But since $\Delta<H$, then, $\Delta<\lambda_S$. I.e. within the limits of the disseminating volume (on sites $-\Delta<Z<0$ and $H-\Delta<Z<H$) the multiplier $e^{-\frac{\mu_0+\mu_S}{\cos\alpha}}$ changes insignificantly.

For sites of an entrance and an exit we shall receive

$$B(Z) = C\left.\frac{\partial V(Z)}{\partial Z}\right|_{\text{вх}}\cdot e^{-\frac{\mu_0+\mu_S}{\cos\alpha}\chi} \text{ - an entrance} \tag{5}$$

and

$$B(Z)=C\frac{\partial V(Z)}{\partial Z}\bigg|_{вых}\cdot e^{-\frac{\mu_0+\mu_S}{\cos\alpha}H}\cdot e^{-\frac{\mu_0+\mu_S}{\cos\alpha}\chi} \text{ - an exit} \tag{6}$$

Introduction of new variable χ (equal Z-0 for an entrance and Z-X for an exit) allows to analyze in common behavior of function *B(z)* on sites of an entrance and an exit since both in that and in other case χ changes from –Δ up to 0. Since χ differs from Z on a constant (0 - on an entrance, H - on an exit) the derivative on Z will be equal to a derivative on χ.

Character of behavior of functions $B(\chi)_{in}$ and $B(\chi)_{out}$ will be identical, and will be described by expressions

$$\begin{cases} B(\chi)\big|_{in} = C\dfrac{\partial V(\chi)}{\partial\chi}e^{-\frac{\mu_0+\mu_S}{\cos\alpha}\chi} \\ B(\chi)\big|_{out} = C'\dfrac{\partial V(\chi)}{\partial\chi}e^{-\frac{\mu_0+\mu_S}{\cos\alpha}\chi} \end{cases} \tag{7}$$

where
$$C'=C\cdot\exp\left[-\frac{\mu_0+\mu_S}{\cos\alpha}H\right] \tag{8}$$

Hence, the behavior of function *B(z)* is described by behavior of a derivative $\frac{\partial B(Z)}{\partial Z}$ and a multiplier $\exp\left[-\frac{\mu_0+\mu_S}{\cos\alpha}\chi\right]$.

Let's consider two essentially distinguished cases.

1. <u>Zero approximation</u>. Easing at movement of a primary beam with energy E_0 within the limits of the disseminating volume V can be neglected.

In this case it is possible to write down

$$\begin{cases} B(\chi)\big|_{in} = C\dfrac{\partial V(\chi)}{\partial\chi}\bigg|_{in}\left(1-\dfrac{\mu_0+\mu_S}{\cos\alpha}\chi\right) \\ B(\chi)\big|_{out} = C'\dfrac{\partial V(\chi)}{\partial\chi}\bigg|_{out}\left(1-\dfrac{\mu_0+\mu_S}{\cos\alpha}\chi\right) \end{cases} \tag{9}$$

And in view of trifle $\frac{\mu_0+\mu_S}{\cos\alpha}\chi<<1$, we shall receive

$$\begin{cases} B(\chi) = C\dfrac{\partial V(\chi)}{\partial\chi}\bigg|_{in} - input \\ B(\chi) = C'\dfrac{\partial V(\chi)}{\partial\chi}\bigg|_{out} - output \end{cases} \tag{10}$$

I.e. for zero approximation the behavior of function *B(χ)* on sites of an entrance and an exit completely repeats behavior of a derivative $\frac{\partial V(\chi)}{\partial\chi}=\frac{\partial V(Z)}{\partial Z}$.

For geometry of Fig. 1. $\frac{\partial V(\chi)}{\partial\chi}\Big|_{in}$ and $\frac{\partial V(\chi)}{\partial\chi}\Big|_{out}$ also will describe speed of change accordingly, an entrance part of the disseminating volume (site GFE).

Let's consider in more details change of the disseminating volume on an entrance and an exit from a layer of heterogeneity (Fig. 2.). The area of the entering volume (triangle ABF) is equal $V_{in}(\chi)=\chi^2 tg\alpha$, hence, a derivative

$$\frac{\partial V(\chi)}{\partial\chi}=2\chi\cdot tg\alpha \tag{11}$$

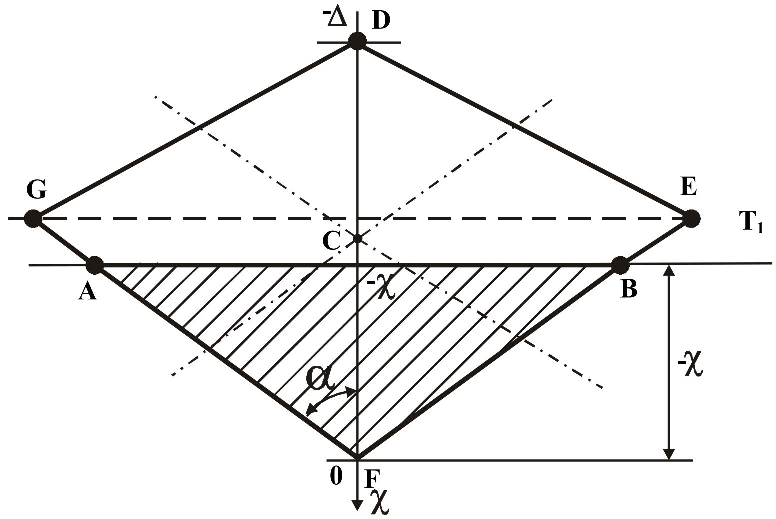

Fig. 2. Geometry of transition of the disseminating volume through border T_1

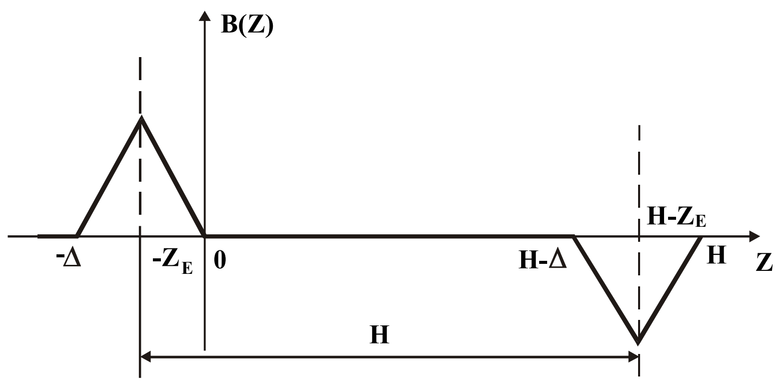

Fig. 3. Character of change of scanning function *B(z)*.

I.e. it is linear growing function on a site from F up to E and decreasing function on a site from E up to D.

At an exit of the disseminating volume from heterogeneity (transition through border T_2) the signal will be determined by remaining part $V_{ост}(\chi)=V_0-V_{вх}(\chi)=V_0-\chi^2 tg\alpha$, where $V_0=const$, equal to full size of volume $\frac{\partial V_{ост}(\chi)}{\partial\chi}=-\frac{\partial V_{вх}(\chi)}{\partial\chi}$

Thus, at use of zero approximation easing in limits DV, function *B(z)* looks like, submitted on Fig. 3. Maximum *B(z)* will correspond to transition through forward border of line GE, and a minimum - at transition of same line through back border.

The distance between maxima and minima of function *B(z)* will be equal to the size of roughness H, since $Z_{max} = -Z_E$; $Z_{min} = H-Z_E$; $Z_{max}-Z_{min} = H$.

From the carried out analysis it is visible, that the distance between a maximum and a minimum of scanning function *B(z)* can be used as the dimensional characteristic of heterogeneity.

The reasons executed above are fair at higher disseminating ability of heterogeneity in comparison with disseminating ability of a material of object. All laws are kept, and when disseminating ability of heterogeneity will be less than disseminating ability of other material. Character of behavior of scanning function

B(z) will differ only that at transition through forward border *B(z)* will have a negative sign, and at an exit from heterogeneity - positive. The distance between a maximum and a minimum will define still extent of heterogeneity H.

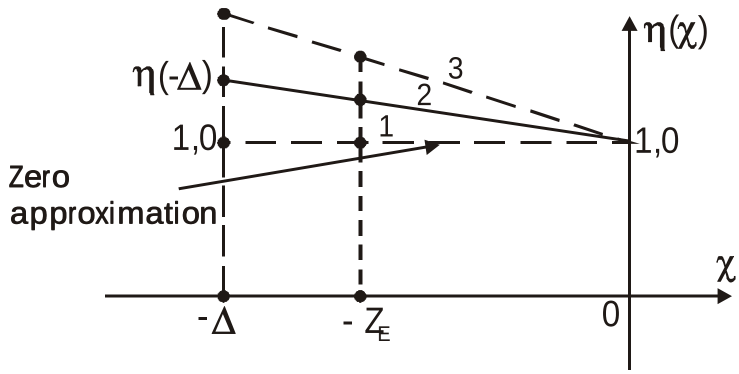

Fig. 4. Change "deforming" multiplier $\eta(\chi)$

2. <u>The first approximation</u>. Easing of primary and absent-minded radiations in limits V to neglect it is impossible. Such situation takes place when the sizes of volume V(Δ) cannot be made much less H. In this case the second composed in brackets in (9) to neglect it is impossible. Since the back border of heterogeneity all the same is necessary for seeing $\frac{\mu_0+\mu_S}{\cos\alpha}\chi$, cannot be >>1, but it is comparable with it. From (9) it is visible, that as against zero approximation, function *B(z)* contains a multiplier $\eta=1+\frac{\mu_0+\mu_S}{\cos\alpha}\Delta$ which changes in limits DV from $1+\frac{\mu_0+\mu_S}{\cos\alpha}\Delta$ up to 1. This multiplier represents the linear function submitted on Fig. 4. and it deforms function $\frac{\partial V(\chi)}{\partial\chi}$. The degree of deformation will depend on a degree of attenuation in limits V. The zero approximation considered above corresponds to a line 1 $(\eta=1)$. Presence of easing is described by lines 2 and 3, and the more easing, the more steepness of a line 3.

Let's consider, how the multiplier $\eta(\chi)$ deforms function *B(χ)*.

<u>The right slope</u>. At movement χ from 0 to the left values $\frac{\partial V(\chi)}{\partial\chi}$ in each point are multiplied on escalating factor that results in increase in values B(χ) all in the greater degree in process of movement from χ=0 to the left.

In result the right front of function *B(χ)* is bent, caving in "inside of" (Fig. 5).

<u>The left slope</u>. As in a point $\chi=-\Delta X$ function *B(χ)* accepts value 0 rise of the right slope, i.e. sizes B(χ=-Z_E), will always result in increase in a steepness of the left slope.

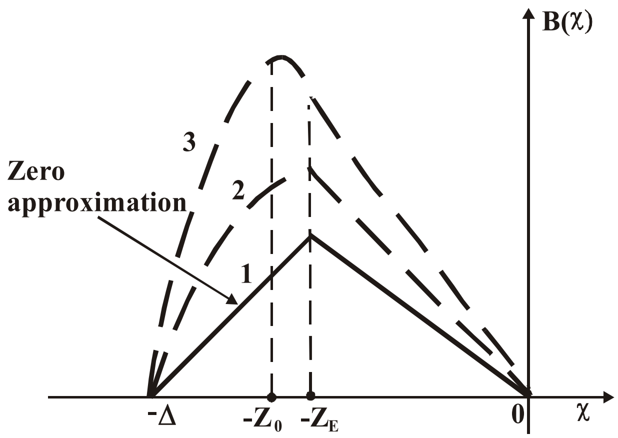

Fig. 5. Deformation of scanning function $B(\chi)$multiplier $\eta(\chi)$

Besides the sites closer worth to - Δ, are multiplied on great values $\eta(\chi)$. Therefore the left slope of function $B(\chi)$ also is bent, but becomes convex. Behavior $B(\chi)$ in immediate proximity to the left of $-Z_E$ depends on a ratio of growth rate of factor $\eta(\chi)$ and reduction $\frac{\partial V(\chi)}{\partial \chi}$.

Displacement of a maximum on an entrance will be same, as well as displacement of a minimum on an exit from a layer of heterogeneity. Therefore the difference of coordinates of a maximum and a minimum of function $B(z)$ also will be equal H and, hence, can be used as a measure of the linear sizes of heterogeneity.

For calculation of displacement of a maximum concerning point Z_E (position of point Z_0) it is necessary to calculate a derivative (7). We shall designate $\frac{\mu_0+\mu_S}{\cos\alpha}=\tilde{\mu}$.

Then from (7) it is possible to write down

$$\begin{cases} B(\chi)\big|_{in} = C\dfrac{\partial V(\chi)}{\partial \chi}e^{-\mu\chi} - input \\ B(\chi)\big|_{out} = C\cdot e^{-\mu H}\dfrac{\partial V(\chi)}{\partial \chi}e^{-\mu\chi} - output \end{cases} \quad (12)$$

$B(\chi)\big|_{вых}$ changes just as and $B(\chi)\big|_{вх}$, but on amplitude in $\exp[-\tilde{\mu}H]$ time less and has a negative sign. Therefore displacement of a minimum on an exit and a maximum on an entrance will be identical always. Hence the difference of coordinates of a maximum and a minimum of function $B(Z)$ always gives thickness.

For calculation of absolute value of shift within the limits of the disseminating volume it is necessary to calculate χ_0 (Fig. 5), i.e. value χ, at which

$$\left[\frac{\partial V(\chi)}{\partial \chi}e^{-\tilde{\mu}\chi}\right]'=0 \quad (13)$$

This condition takes place for differentiated function. Product (9) is defined by two components: by derivative $\frac{\partial V(\chi)}{\partial \chi}$ and exponent (Fig. 6). Derivative $\frac{\partial V(\chi)}{\partial \chi}$ (the line 1) has an excess in a point $-Z_E$.

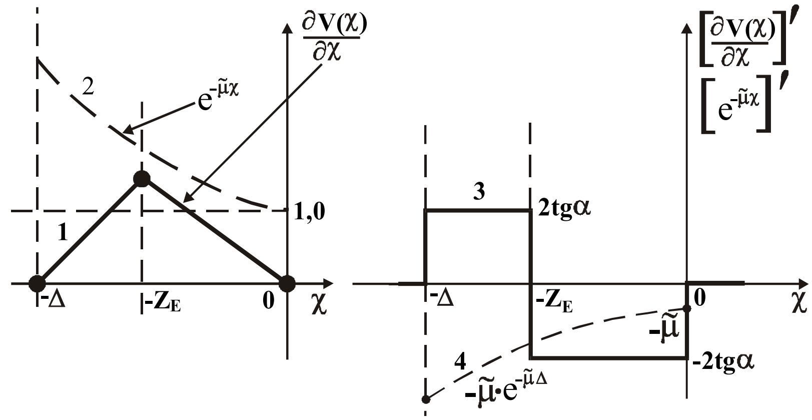

Fig. 6. Behavior of derivatives $\left[\frac{\partial V(\chi)}{\partial\chi}\right]$ (a line 1) and $\left[\exp(-\tilde{\mu}\cdot\chi)\right]'$ (a line 4) within the limits of the disseminating volume.

However if function $\left[\frac{\partial V(\chi)}{\partial\chi}\right]'$ changes a sign in a point $-Z_E$ from plus for a minus this point is a point of a maximum. The factor $e^{-\tilde{\mu}\chi}$ looks like 2 (Fig. 6.). Its derivative is equal $-\mu\cdot\exp(\tilde{\mu}\chi)$. It is a curve 4.

The full derivative (13) and a condition of an extremum will look like

$$\left[\frac{\partial^2 V(\chi)}{\partial\chi^2}-\tilde{\mu}\frac{\partial V(\chi)}{\partial\chi}\right]\cdot e^{-\tilde{\mu}\chi}=0 \tag{14}$$

Or

$$\frac{\partial^2 V(\chi)}{\partial\chi^2}=\tilde{\mu}\frac{\partial V(\chi)}{\partial\chi} \tag{15}$$

With the account (9), (11) it is possible to write down

$$\frac{\partial V(\chi)}{\partial\chi}=\begin{cases}2(\chi+\Delta)tg\alpha & for-\Delta<\chi<Z_E\\ 2\chi tg\alpha & for-Z_E<\chi<0\end{cases}$$

$$\frac{\partial^2 V(\chi)}{\partial\chi^2}=\begin{cases}2tg\alpha & for-\Delta<\chi<Z_E\\ -2tg\alpha & for-Z_E<\chi<0\end{cases}$$

Corresponding curves 1 and 3 are resulted on Fig. 6.

The condition (15) can be considered for three special cases:

1. <u>The big energy, small density</u>. In this case easing is not enough $\tilde{\mu}\approx 0$ and we receive the zero approximation considered above. The point of position of an extremum is a point $-Z_E$ since in it $\frac{\partial^2 V(\chi)}{\partial\chi^2}$ has break.

2. <u>Small energy, the big density</u>. Easing is great also radiation does not pass in heterogeneity. Dispersion will occur only in a thin layer on border T_1. In this case volume V degenerates in line AB (Fig. 2) which length is equal $V(\chi)=2\chi tg\alpha\cdot$ Accordingly scanning function $\frac{\partial V(\chi)}{\partial\chi}$ degenerates in a line of 2 Fig. 7, and the derivative $\left[\frac{\partial V(\chi)}{\partial\chi}\right]'$ degenerates in a combination three delta - pulses (6 Fig. 7), not

bearing{carrying} the information on heterogeneity owing to absence of dispersion on border T1. The information on the bottom border of heterogeneity T_2 also is absent, since radiation does not pass in it.

3. Intermediate energy and density. At all energies and densities $\tilde{\mu}>0$ essentially positive size, $\frac{\partial V(\chi)}{\partial \chi}$ also in all area from up $-\Delta$ to 0 is positive. Therefore $\frac{\partial V(\chi)}{\partial \chi}\tilde{\mu}$ should be positive. Hence, (15) it is possible to search for the decision of the equation only in the field of values χ, where $\frac{\partial^2 V(\chi)}{\partial \chi^2} > 0$. And it is area $-\Delta < \chi < -Z_E$. Hence, displacement of a maximum of function *B(χ)* can take place only aside values χ, for which $|\chi| > |Z_E|$, i.e. in area from up $-\Delta$ to $-Z_E$.

The second derivative on this site is a constant equal $\tilde{C}$ (a line 2, Fig. 7).

Therefore the condition of an extremum can be written down

$$\left.\frac{\partial V(\chi)}{\partial \chi}\right|_{extrem} = \frac{\tilde{C}}{\tilde{\mu}} \qquad (16)$$

It is necessary to exclude a point from this condition $-Z_E$, since in it there is a break and it works in case of 1.

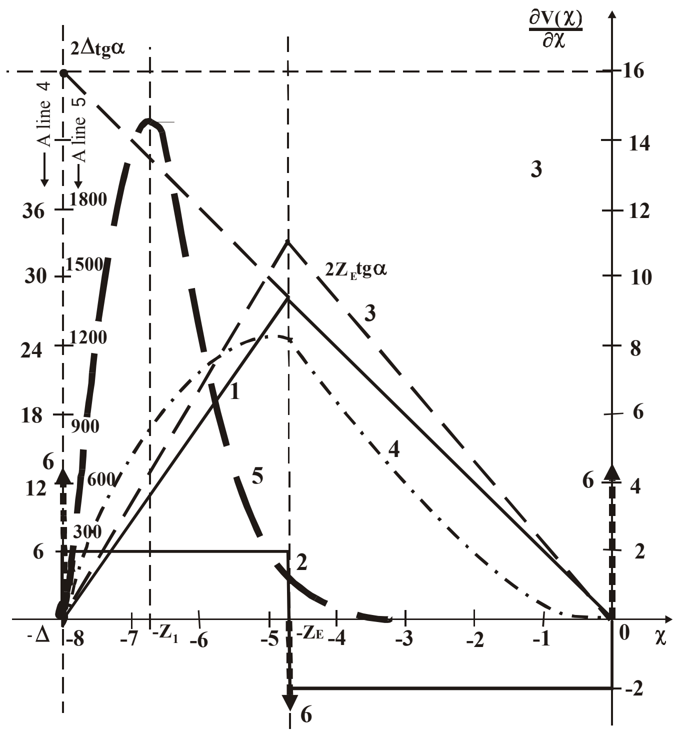

Fig. 7. Geometry of the disseminating volume: Δ=8mm., α=45^0.

Line 1 $\rightarrow \frac{\partial V}{\partial \chi}$; line 2 $\rightarrow \frac{\partial^2 V}{\partial \chi^2}$; line 3 $\rightarrow \frac{\partial V}{\partial \chi} \cdot e^{-\mu \cdot \chi}$ ($-\tilde{\mu}\chi$ - it is not enough); line 4 $\rightarrow \frac{\partial V}{\partial \chi} \cdot e^{-\mu \cdot \chi}$ ($-\tilde{\mu}\chi$ - critical); line 5 $\rightarrow \frac{\partial V}{\partial \chi} \cdot e^{-\mu \cdot \chi}$ ($-\tilde{\mu}\chi$ - it is great)

At all smaller $\tilde{\mu}$ the extremum is in the field of the maximal width of the disseminating volume.
At big the $\tilde{\mu}$ point of an extremum will be displaced more to the left -Z_E. As special a point it is possible to take a point -Z_I, in which $\tilde{\mu} = 1mm^{-1} = 0{,}1sm^{-1}$.
If as a condition to accept the requirement of the control of thickness up to 40mm, what $\tilde{\mu} \le \frac{3}{4} = 0{,}75sm^{-1}$.
Let's estimate quantitative changes of the form of function $B(\chi)$ on sites of an entrance and an exit.

At the sizes accepted above and geometry, gives the values submitted in tab. 1 and on Fig. 7.

Table 1

χ, mm	-8	-7	-6	-5	-4,7	-4	-3	-2	-1	0
$\frac{\partial V}{\partial \chi}$	0	2,5	5,6	8,6	9,4	8	6	4	2	0
$B(\chi)$	Do critical attenuation, $\tilde{\mu}$=0,2sm^{-1}									
	0	3,5	7,0	9,6	10,8	9,2	6,5	4,4	2,2	0
	Critical attenuation, =2,0sm^{-1}									
	0	12,2	21,8	24,8	25,0	18,5	12,0	6,0	2,0	0
	Later attenuation, =10sm^{-1}									
	0	2120	1420	410	240	80	14,8	5,4	0	0

From them follows, that at the big factors of attenuation extrema of scanning function $B(\chi)$ are displaced concerning geometrical center DV aside its top half, however distance between them, at preservation of form DV, do not change.

Conclusions

The analysis of behavior of scanning function *B(z)* has shown:

1. In a working range энергий and density the developed scanning function *B(z)* provides measurement of the sizes of heterogeneity irrespective of conditions of the control, i.e. energy of probing radiation and density of a material of heterogeneity;

2. A measure of the linear sizes of heterogeneity is the distance between positive and negative extrema *B(z)* at moving the disseminating volume through heterogeneity;

3. The form of scanning function *B(z)* is deformed on sites of transition through borders of heterogeneity due to easing primary and absent-minded radiations within the limits of the disseminating volume. This deformation can make up to 15% in comparison with the geometrical form;

4. In a range «precritical attenuations» extrema are not displaced and coincide with position of the maximal width of the disseminating volume;

5. At an entrance of the disseminating volume in a product from air the first maximum of function *B(z)* can be used for definition of border of all product that allows to define the form of object of the control;

6. At the control of the materials close to homogeneous, to one the primary goals measurement of thickness of a product is. The developed method and algorithm provide an error of such measurement no more in a range энергий from 20 up to 1000K and for materials with density up to 2,7г/sm^3.

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